Constraining Cosmological Topology via Highly Luminous X-ray Clusters

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ABSTRACT

The topology of the observable Universe is not yet known. The most significant observational sign of a non-trivial topology would be multiple images ("ghosts") of a single object at (in general) different sky positions and redshifts.

It is pointed out that the previous search by Gott (1980) for ghost images of the Coma cluster can be extended by using highly X-ray luminous clusters of galaxies. This is likely to be more efficient than with other astrophysical objects viewable on these scales since (1) X-ray clusters would be at least as easy to identify if viewed from other angles as any other objects and (2) the X-ray emitting thermally heated gas is likely to be simpler than for other objects.

Possibilities that the highly luminous cluster RXJ 1347.5-1145 ($z = 0{\cdot}45$) has a "ghost image" at lower redshift are analysed. It is noted that RXJ 1347.5-1145, the Coma cluster and the cluster CL 09104+4109 form nearly a right angle ($\approx 88°$) with arms of nearly identical length ($970h^{-1}$ and $960h^{-1}$ Mpc respectively) for $\Omega_0 = 1, \lambda_0 = 0$ curvature ($h \equiv H_0/100\mathrm{km\ s}^{-1}\ \mathrm{Mpc}^{-1}$). This is a clue that the three clusters could be ghost images of one and the same cluster, for a hypertoroidal topology. However, several arguments are presented that this relation is not physical.

Key words: methods: observational — cosmology: observations — galaxies: clusters: individual (RXJ 1347.5-1145) — galaxies: clusters: individual (Coma) — X-rays: galaxies

1 INTRODUCTION

From the beginning of modern cosmology, it has been realised that both the curvature and topology of the observable Universe may be nontrivial (e.g., de Sitter 1917; Lemaître 1958), but the fact that objects at cosmological distances are seen in our past time cone has meant that constraints on topology suffer astrophysical complications just as do the constraints on curvature and on the Hubble constant.

The empirical tradition in cosmology requires that observations are used to constrain both curvature and topology. Theoretical indications that the topology should be nontrivial are not lacking, however. Indeed, observational indications are that the curvature of the Universe is either negative or zero, implying that for a trivial topology the volume would be infinite. This implies that the requirement of quantum gravity with a "no-boundary" boundary condition (e.g., Hawking 1984; Zel'dovich & Grishchuk 1984) that the Universe be compact is not satisfied by a trivial (simply connected) topology. A multi-connected negatively curved (or flat) universe of finite volume provides such compactness.

A completely independent argument for compactness is from unified particle physics theories which imply that most of 10 or 11 dimensions of the fundamental "strings" are "compactified". If the volume of the Universe were infinite, it would seem somewhat arbitrary that the three space dimensions are an exception to the compactification of the other dimensions. (This argument is known as "Dimensional Democracy, but where some dimensions are more equal than others", Starkman et al. 1996; Orwell 1945.) Multi-connectedness can save a finite, non-positively curved universe.

A third argument comes from inflationary theory for negatively curved universes. Observational difficulties with an $\Omega_0 = 1$ universe have stimulated inflation models which predict that $\Omega_0 + \lambda_0 < 1$ (Gott 1982, 1986; Sasaki et al. 1993; Linde 1995; for consequences on the cosmic microwave background, see Ratra & Peebles 1994; Bucher & Goldhaber 1995). However, inflation resulting in negative curvature at the present epoch requires fine-tuning the initial conditions, which is provided in these models by an earlier period of inflation. In that case the inflationary potential is designed in order to yield such "double inflation". On the

other hand, multi-connectedness in a negatively curved universe implies chaoticity of geodesics, and so naturally provides the smooth initial conditions for a single period of inflation which results in observable curvature at the present (Cornish et al. 1996).

Lachièze-Rey & Luminet (1995) present an extensive introductory review of how one can understand nontrivial topology in a standard hot big, bang (Friedmann-Lemaître-Robertson-Walker) Universe, examples of many of the known (orientable) topologies possible, and of observational efforts to measure or constrain the topology.

However, the reader is reminded here of the simplest example of a three-dimensional topology, for trivial curvature—the hypertorus. One can think of the entire Universe as a cube, of side length at least several hundred or thousand Mpc, of which opposite faces are physically identified. A particle such as a photon may travel between two points in space, e.g., from object to observer, by a direct route "within" the cube, or may "pass through" the faces of the cube several times in travelling from one point to the other. Hence, many "ghost" images at (in general) differing celestial positions and (in general) differing redshifts (due to differing light travel time) would be seen. Of course, in proper coordinates, this cube expands in size according to the scale factor, so that detection of non-trivial topology has to be done in comoving coordinates.

This embedding within infinite, flat, three-dimensional space is only an aid to our intuition—the space outside the cube does not have physical significance, although it is useful mathematically. The infinite space (or a hyperbolic or elliptic infinite space for other curvatures) is termed the "universal covering space"; the cube (or in other cases other polyhedra of which faces are identified in some way) is termed the "fundamental polyhedron". Following Lachièze-Rey & Luminet 1995, the shortest distance from an object to any of its "ghost" counterparts is α, and the largest distance from an object to an adjacent ghost (where there is an adjacent ghost for each face of the fundamental polyhedron) is β.

The most direct way of attempting to detect the topology of the Universe is by searching for individual ghost images of known objects in the universal covering space. That is, objects observed in the past time cone are assumed to exist in a simply-connected universe, at distances they would have (assuming zero peculiar velocity) within the simply-connected spatial hypersurface at $t = t_0$. This covering space would contain many copies of the fundamental polyhedron. Constraints obtained by this technique (e.g., Sokolov & Shvartsman 1974; Gott 1980; Fagundes 1985, 1989; Fagundes & Wichoski 1987) give constraints on α, β to a few tens or hundreds of Mpc or else find ghost image candidates statistically consistent with chance coincidences.

Several recent attempts have been made to use the essentially two-dimensional slice of the covering space which temporally corresponds to the period of recombination, i.e., which spatially is the surface of last scattering observed as the cosmic microwave background (CMB) by the COBE satellite (Stevens et al. 1993; Starobinsky 1993; Jing & Fang 1994; de Oliveira Costa & Smoot 1995). These methods rely upon our present understanding of the local physics at the redshift of last scattering and require various assumptions, e.g., assume a toroidal topology, and claim limits to a few thousand Mpc. An interesting advance in techniques of finding candidate manifolds, of hyperbolic (negative) curvature, is that of Fagundes (1996), in which possible identifications between CMB cold spots and galaxy superclusters are used to predict expected positions (α, δ, z) of further ghosts of superclusters. Observational confirmation or refutation of such candidate manifolds is obviously straightforward.

Three topology-independent methods either for three-dimensional catalogues of objects spread over large scales (Lehoucq et al. 1996; Roukema 1996) or for the CMB (Cornish et al. 1997) now provide alternatives to the search for individual ghosts and the CMB methods cited above.

However, the subject of this paper is an extension of the traditional and simplest method, the search for ghost images of individual objects. The explanation of why X-ray clusters are probably the best "topology standard candle" available on scales of about $1000h^{-1}$ Mpc is presented in §2, cases of specific clusters are discussed in §3 and a summary is given in §4.

For brevity of discussion, and because of the theoretical motivations for a finite volume universe, the use below of the terms "multi-connected" and "non-trivial topology" should be taken to imply the finite volume cases only unless otherwise specified.

For reference, the reader should be reminded that the horizon is at $6000h^{-1}$ Mpc from the observer, and the horizon diameter is $12000h^{-1}$ Mpc. (Except where otherwise stated, distances are quoted in comoving units in an $\Omega_0 = 1, \lambda_0 = 0$ universe and $h \equiv H_0/100 \text{km s}^{-1} \text{Mpc}^{-1}$ is explicitly indicated; cluster luminosities and masses implicitly include the assumptions that $\Omega_0 = 1, \lambda_0 = 0, h = 0.5$.)

2   X-RAY CLUSTERS AS TOPOLOGY STANDARD CANDLES

The Einstein, ASCA and ROSAT satellites have shown that many galaxy clusters contain hot, bremsstrahlung, X-ray emitting gas. This is in fact the case for nearly all of the richest (Abell richness class $R > 2$) clusters, and those not detected may be undetected only due to the flux limit of the surveys (Ebeling et al. 1993).

The richest clusters therefore provide what is probably the best "topology standard candle" available on scales of about $1000h^{-1}$ Mpc: (1) clusters should look fairly similar—in X-rays or optically in the statistical distribution of galaxies—from any direction; (2) once the (rich) cluster exists, evolution of the X-ray emitting gas is likely to be only possible in one direction—to greater mass and/or greater mass concentration—the X-ray luminosity could probably not decrease by more than about a factor of two.

These two properties are ideal for searching for ghosts. Property (1), isotropy, is useful since except in special cases, ghosts are likely to be viewed from different angles, so the main objects visible at large redshifts, quasars, are going to be much fainter when seen from many angles (according to unified models of active galactic nuclei they would be seen as, e.g., Seyfert galaxies). This is why Lehoucq et al.'s (1996) statistical method is unlikely to be useful for quasars, so Roukema's (1996) method of searching for individual configurations was developed.

Property (2), the evolutionary constraint, is equally useful. Up to the redshift of rich cluster formation, probably

$z \gtrsim 1$, the rarity of the largest clusters in a simply-connected universe should imply that surveys to larger and larger volumes should yield successively rarer and higher mass clusters, simply since the higher the mass of a cluster the rarer it should be.

On the other hand, a multi-connected universe of finite size smaller than the observed volume must contain a cluster of maximum mass (for precise enough bins in mass determination), which forcibly has at least one ghost image in the copy of the fundamental polyhedron which includes the observer. Ghosts of the maximum mass cluster in copies of the polyhedron at higher redshifts are likely to be of identical or lower mass, if it is accepted that the cluster can only gain in mass once formed.

Hence, in a simply-connected universe, more and more massive (and rare) clusters should be seen to successively larger redshifts (up to the formation epoch of such clusters), while in a small enough (finite volume) multi-connected universe, once *the* most massive cluster has been seen, no clusters of greater mass should be seen at higher redshifts.

In other words, property (2) not only implies that X-ray emitting (rich) clusters would in principle be possible to see as ghost images, but the existence at a "high" redshift of a single such cluster which is much more massive than all the others at lower redshifts provides a strong constraint *against* multi-connectedness up to the redshift of the cluster. The key elements of this constraint are merely gravity and conservation of mass.

2.1 Evolution

What are the possibilities for evolution of the gas component, which generates the X-ray luminosity?

This is likely to depend mostly on the mass of the cluster. Observational links between the X-ray luminosity of a cluster, L_X, and the gas temperature, T, (e.g., Edge & Stewart 1991; Henry & Arnaud 1991), combined with a theoretical isothermal potential $M - T$ relation (where M is the virial mass; e.g., Evrard et al. 1996), indicate that $L_X \sim M^2$. While this relationship is not evolutionary, it at least suggests that if the mass of a cluster changes, its luminosity is likely to increase strongly.

Direct attempts to measure L_X evolution with redshift z have had diverse results (Gioia et al. 1990; Edge et al. 1990; Henry et al. 1992; Luppino & Gioia 1995; Oukbir et al. 1997), but Ebeling et al. (1996b; 1997) point out that several of the analyses may have suffered selection effects, and find no significant evolution for X-ray selected ROSAT clusters to $z \lesssim 0\cdot3$. Even if the suggestions of "negative" evolution were correct, this would mean that L_X increases with time, strengthening the usefulness of clusters as topological standard candles.

These results seem reasonable theoretically. Since the gas is a much larger component (by mass) than the galaxies, and given the time, length and mass scales involved[⋆], there is little possibility that the hot gas can collapse mostly into galaxies (or H_2 clouds) or be blown out of the cluster over several Gigayears. So the gas must remain in the cluster, and since, at least to a good first approximation (apart from

cooling flows), is in isothermal equilibrium in the potential well of the cluster, the density distribution is unlikely to change by much. Since the X-ray emission is believed to be bremsstrahlung emission due to particles accelerating in a potential well of $\sim 10^{14-15} M_\odot$, it seems physically unlikely that this emission could be suppressed by large factors.

The most physically reasonable scenarios for evolution of the gas distribution (and the overall mass distribution) would be for the density distribution to become more centrally concentrated or for the mass to increase somewhat due to gas (and galaxies) still infalling into the cluster. These cases would agree with the EMSS data analysis (Gioia & Luppino 1994; Oukbir & Blanchard 1997) suggesting increase of L_X with time. Interaction with another large cluster might possibly loosen up the central concentration (the core), but such a case is likely to be both rare and easy to spot.

Of course, a significant fraction of cluster X-ray luminosity often comes from what are (usually believed to be) cooling flows (e.g., Fabian & Crawford 1995). Over a Hubble time a cooling flow could feed a few percent of the cluster mass into the centre of the cluster, not significantly changing the (approximately) isothermal gas distribution. If a cooling flow ceased between the epochs of two ghost images, the X-ray luminosity could decrease by as much as a factor of two, lowering the chances of detection above a given flux limit. However, this would again (by energetic considerations) require collision with another large cluster.

So, it is difficult to see how ghosts of "high redshift" X-ray clusters at lower redshifts could be too faint to have been detected.

3 DOES RXJ 1347.5-1145 HAVE ANY GHOST IMAGES?

Observational (or observationally deduced) parameters of RXJ 1347.5-1145 Schindler et al. (1996) and of some of the most highly luminous X-ray clusters which could be candidates for a ghost image of RXJ 1347.5-1145 are listed in Table 1.

These luminosities (in the $0\cdot1 - 2\cdot4$ keV band, as illustration) are shown graphically in in Fig. 1. RXJ 1347.5-1145 is clearly exceptional relative to the other massive clusters.

The argument presented in §2 that brighter (or more massive), hence, rarer clusters should be seen to higher and higher redshifts (up to the cluster formation epoch, and assuming little evolution) is also illustrated in this Figure. From the Schechter (1976) function fit to the ROSAT Brightest Cluster Sample of Ebeling et al. (1997) it is straightforward for a given redshift to predict the luminosity such that clusters of that luminosity become "common" enough (one occurrence) in the increased available volume to be visible in surveys. This "brightest observable cluster in a finite volume" statistic is shown both for a universe of trivial topology and for a universe of non-trivial topology with a fundamental polyhedron diameter, for illustration purposes, of $600h^{-1}$ Mpc.

If RXJ 1347.5-1145 did not exist, previous astrophysical object based limits on the diameter of the fundamental polyhedron (e.g., Lehoucq et al. 1996) would be consistent with the existence of the other bright clusters represented. One

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⋆ Remember that $1000\,\mathrm{km\,s^{-1}} \approx 1\mathrm{Mpc\,Gyr^{-1}}$.

Table 1. Basic data on highly luminous X-ray clusters, assuming $\Omega_0 = 1, \lambda_0 = 0, H_0 = 50$ kms^{-1} Mpc^{-1}. M_r indicates total mass to r Mpc in units of $10^{14} M_\odot$; L_X(W) is X-ray luminosity in units of 10^{44} erg/s in band W=1 $(0.1 - 2.4$ keV), W=2 (2-10 keV) or W=b ("bolometric"), T, r is temperature T in keV within a radius r. Luminosities for A2163, Coma, A1835 and Perseus are from Ebeling et al. (1996a); other references are Elbaz et al. (1995), Schindler et al. (1996), Briel et al. (1992), Hall et al. (1997), Fabian & Crawford (1995) and Allen et al. (1996) as indicated by abbreviations.

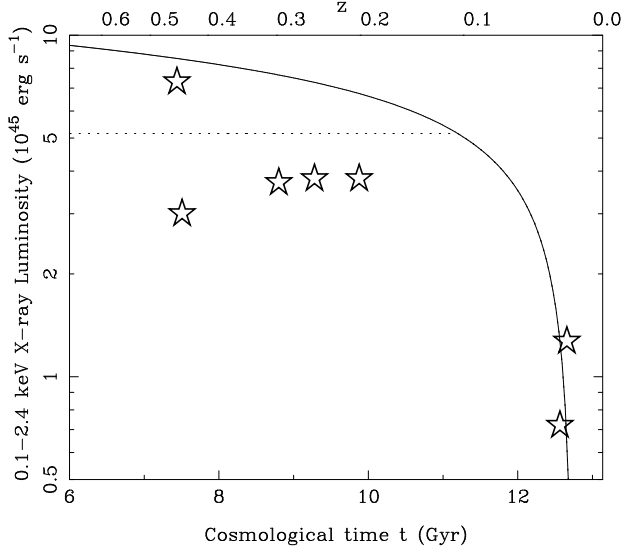| Object	α	δ	z	$M_{1.5}$	M_3	$M_{4.6}$	M_5	$L_X(1)$	$L_X(2)$	$L_X(b)$	T, r	ref.
A2163	$16^h 15^m.75$	$-6°09'$	0.201			46^{+4}_{-15}		38	41		$14.6 \pm 1, 4.2'$	Elb95
RXJ 1347.5 − 1145	$13^h 47^m.5$	$-11°45'$	0.451		17			73	76	200	$9.3 \pm 1, 7'$	Sch96
Coma cluster	$12^h 57^m$	$28°15'$	0.023		15 ± 5		18 ± 6	7.2	8.8		$8 \pm 3, 16'$	Br92
CL 09104 + 4109	$09^h 10^m.5$	$41°09'$	0.442					30			$11.4^{+\infty}_{-3.2}$	H97, FC95
CL 1821 + 643	$18^h 21^m$	$64°03'$	0.297					37				H97
A1835	$14^h 01^m$	$02°53'$	0.252	9				38	43		$9.5 \pm 2, 6'$	A97
Perseus cluster	$03^h 15^m.3$	$41°20'$	0.018					12.7	11.9		6.7 ± 2	

Figure 1. X-ray luminosities of several of the brightest clusters shown against cosmological time (values in Table 1). A solid curve shows the brightest luminosity at redshift z to which the observed $0.1 - 2.4$ keV X-ray luminosity function (parametrised as a Schechter function; values and units of Ebeling et al. 1997) predicts a number density of one object per the total observable volume to z, assuming a trivial topology. (One-third of the sky is assumed unobservable due to the galactic plane.) The dotted curve shows the same brightest luminosity statistic for a universe of non-trivial topology, with a fundamental polyhedron diameter of $600 h^{-1}$ Mpc. ($\Omega_0 = 1, \lambda_0 = 0, h = 0.5$ are used.)

might be encouraged to search for signs of identity among the four $z > 0.2$ clusters shown in Fig. 1, or to look for ghosts of these objects at higher redshifts. However, RXJ 1347.5-1145 does exist, and its discovery at a moderately high redshift does not imply that clusters were brighter in the past, it is simply consistent with being a rare cluster requiring a large search volume, under the assumptions of a trivial topology of the Universe and no significant luminosity evolution!

Nevertheless, we do consider some possibilities of ghost images of RXJ 1347.5-1145 below.

3.1 Is Coma a Ghost of RXJ 1347.5-1145?

Could the Coma cluster be a low redshift image of RXJ 1347.5-1145? This would obviously be of tremendous usefulness in understanding both cluster and galaxy evolution, given the huge time lag between the two images.

Both clusters have two large "central dominant" (cD) galaxies at their centres; the total masses are identical within the uncertainties [Briel et al.'s (1992) ROSAT mass estimate for Coma implies a total mass of $(4 - 10) \times 10^{14} M_\odot$ to $1 h_{50}^{-1}$ Mpc) and $(10 - 20) \times 10^{14} M_\odot$ to $3 h_{50}^{-1}$ Mpc; Schindler et al. give $5.8 \times 10^{14} M_\odot$ and $17 \times 10^{14} M_\odot$ for RXJ 1347.5-1145 to the same radii respectively]; and the gas fraction to a large radius appears lower in Coma, but consistent within the uncertainty.

However, the much preciser X-ray fluxes and core $(r < 500 h_{50}^{-1}$ kpc) surface brightnesses are much higher for RXJ 1347.5-1145 than for Coma. If the ghost of RXJ 1347.5-1145 were to lie as close to us as Coma, without any reduction in the bremsstrahlung luminosity of the hot (and cooling) gas, then it would have a $0.1 - 2.4$ keV X-ray flux of 3.4×10^{-9} erg/s/cm^2, which is six times as bright as the Perseus galaxy cluster and ten times Coma itself. For RXJ 1347.5-1145 and Coma to be ghost images of one another, this would require not only removal of the cooling flow, but also some means of further reducing the hot gas emission by a factor of about ten. Moreover, the core gas would have to be radically redistributed in the $\approx 2.6 h_{50}^{-1}$Gyr (for $\Omega_0 = 1, \lambda_0 = 0$) time lapse between the observed two emission epochs. For this to be done, as suggested above, by collision with another cluster, this second cluster should be visible close to RXJ 1347.5-1145 in the ROSAT pointed observations or should be detectable in moderately deep optical imaging over a 10–15′ field around the cluster.

So it seems difficult to see how Coma could physically be identical to RXJ 1347.5-1145.

3.2 Other Candidates for a Ghost of RXJ 1347.5-1145

The fact that the redshifts of RXJ 1347.5-1145 and CL 09104+4109 are similar provides a time scale argument against these two clusters being physically identical.

Since the redshifts are close, we in fact see CL 09104+4109 at an epoch only $35h^{-1}$ Myr (for $\Omega_0 = 1, \lambda_0 = 0$) later than RXJ 1347.5-1145, since the light travel times are indicated by the redshifts, irrespective of topology. A very IR-luminous quasar is seen in CL 09104+4109, but a strong IR source has not been found in RXJ 1347.5-1145. If the ordering here were the opposite, an estimate of the lifetime of the IR emission $\gg 35$ Myr would make it hard to understand why the emission is not seen in RXJ 1347.5-1145. However, RXJ 1347.5-1145 is the earlier image, so we can only consider the probability that two nearly simultaneous snapshots of the cluster happen to occur before and during the period of IR emission.

What is the likely lifetime of the far-IR emission? Starlight from stars created by the process causing the quasar, e.g., galaxy merging, reradiated in the far-IR is a likely candidate. A typical dynamical time scale for such an event, supposing this involves two typical large galaxies of $10^{12} M_\odot$, is about 200 Myr (e.g., Scoville & Soifer 1991), while the massive stars formed during this period would remain on the main sequence for about this much time afterwards. Cavaliere & Padovani (1988) estimate that quasar lifetimes are likely to be at most about a Gigayear. In either case, in the case of physical identity of RXJ 1347.5-1145 and CL 09104+4109, it would seem to be about a 10% coincidence that we happen to see one ghost image just before the event started and one during the event.

Most of the other candidate ghost images of RXJ 1347.5-1145 listed in Table 1 seem to be of lower luminosity and are at lower redshifts, requiring a reduction in L_X with time in order to be identical with RXJ 1347.5-1145, which as discussed above is physically unlikely. On the other hand, A2163 is at lower redshift than RXJ 1347.5-1145, but is considerably more massive, making it too an unlikely ghost candidate.

3.3 A Ghost in the Plane?

Could a ghost of RXJ 1347.5-1145 be hiding in the galactic plane? The fraction of the soft X-ray flux absorbed by the Galaxy varies from 10–20% at the galactic poles to 95% on the plane (assuming a galactic plane column density of $n_H = 3 \times 10^{22}$ cm^{-2} for $|b^{II}| = 0°$). This problem is even worse if one considers the detected ROSAT count rate which is predominantly at 1 keV where the detected count rate is down by 98% on the plane. At a galactic latitude of $|b^{II}| = 20°$ the fraction of absorbed flux is about 50–60% (with a large variance), so studies in soft X-rays (e.g. Ebeling et al. 1993) are limited to the two thirds of the sky at higher latitudes.

The existence of a ghost in the plane is therefore a possibility, which cannot easily be excluded. However, this is a fundamental limitation to any search for individual ghost images, which affects all other candidate objects.

One future possibility would be for the ABRIXSAS satellite to perform an all-sky survey in hard ($E > 3$ keV) X-

Table 2. Basis vectors $L\mathbf{e}_i$ of candidate manifold, in cartesian equatorial coordinates ($x = r \cos\delta \cos\alpha$, $y = r \cos\delta \sin\alpha$, $z = r \sin\delta$, for radial comoving distance r) in units of h^{-1} Mpc. $\Omega_0 = 1, \lambda_0 = 0$ is assumed. The objects defining these basis vectors are listed (but note that the vectors are modified in order to give an exact cube as the fundamental polyhedron, of side length $L = 962h^{-1}$ Mpc).

Motivation for $L\mathbf{e}_i$	$L\mathbf{e}_i$	x_i	y_i	z_i
Coma to RXJ 1347.5-1145	$L\mathbf{e}_1$	-813	-446	-254
Coma to CL 09104+4109	$L\mathbf{e}_2$	-490	533	633
orthonorm. to vectors 1, 2	$L\mathbf{e}_3$	-153	665	-678

ray bands, in which the galactic plane is substantially more transparent. However, in this case, since it would be difficult to confirm any hard X-ray detected candidate clusters at other wavelengths, a physical test for distinguishing clusters from other sources in the infrared or using the hard X-ray data alone (e.g., from surface brightness profiles) would need to be established from the sources at high galactic latitudes. Whether or not this is feasible remains to be seen.

3.4 A Candidate Multi-connected Manfold

While the "local physics" arguments just presented argue against either Coma or CL 09104+4109 being ghost images of RXJ 1347.5-1145, it is quite exciting to note nevertheless that examination of the three-dimensional positions of these bright clusters yields a geometrical pattern indicative of a candidate multi-connected manifold; specifically, a hypertorus, of which two side lengths are just under 1000 kpc. RXJ 1347.5-1145, the Coma cluster, and the cluster CL 09104+4109 (Hall et al. 1997) form nearly a right angle ($\approx 88°$) with arms of nearly identical length ($970h^{-1}$ and $960h^{-1}$ Mpc respectively) for $\Omega_0 = 1, \lambda_0 = 0$ curvature.

Of course, the CMB analyses cited above specifically concentrate on the case of a hypertoroidal universe, since this is the simplest of non-trivial topologies possible, so it is unlikely that a hypertorus on the scale required for identity of the three clusters has managed to escape attention so far. However, this geometrical configuration is one which has already been searched for among other objects (e.g., quasars, Fagundes & Wichoski 1987), and was found among a very small number of candidate objects, so it is certainly interesting to consider independently of the CMB analyses.

The mapping from the universal covering space to a single copy of the fundamental polyhedron, which in this case is assumed to be a cube (although other possibilities for the third dimension could be considered), is very simple. By forcing the three clusters to form an exact right angle of arms (axes) of identical length, and choosing the third axis to be at right angles to the first two and of the same length, the transformation from the universal covering space to the fundamental polyhedron is simply

$$\mathbf{r}' = [\mathbf{r}.\mathbf{e}_1 (\mathrm{mod}\, L),\ \mathbf{r}.\mathbf{e}_2 (\mathrm{mod}\, L),\ \mathbf{r}.\mathbf{e}_3 (\mathrm{mod}\, L)] \qquad (1)$$

where $\mathbf{r}$ is the (three-dimensional) position of any astrophysical object, $(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$ is the orthonormal basis listed in Table 2, L is the side length of the fundamental cube and $\mathbf{r}'$ is the object's position translated into the fundamental cube
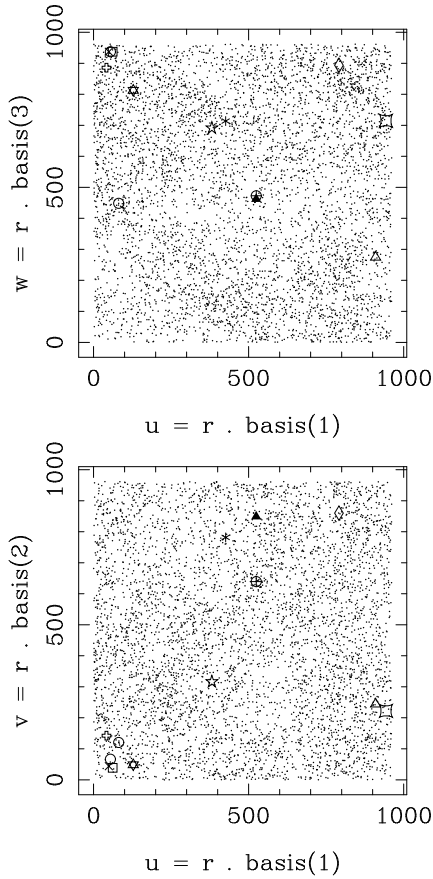
Figure 2. Observed quasars (N=5007) shifted to fundamental cube according to the linear transformation defined by identifying RXJ 1347.5-1145, the Coma cluster and the cluster CL 09104+4109 and supposing the third axis to be perpendicular and of the same length as these identities (Table 2; Eqn 1). The points represent quasars; higher quality topological standard candles plotted are clusters A2163 (asterisk), RXJ 1347.5 (circle), Coma ("x"), CL 09104+4109 (square), CL 1821+643 (triangle); superclusters Ursa Major (Swiss cross) and CrB (Star of David); and six CMB cold spots (other large symbols). The three clusters defining the transformation can be seen lying nearly on top of one another in the bottom-left of the $u - v$ panel and the top-left of the $u - w$ panel.

and expressed in the coordinate system of the orthonormal basis.[†]

Readers can easily use this transformation to check for themselves whether or not catalogues of objects at large redshifts transformed into the (candidate) fundamental cube happen to coincide with one another—as should be the case

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[†] Note to Fortran users: the Fortran $\mathrm{mod}(a,b)$ function should be modified for use with negative values of a.
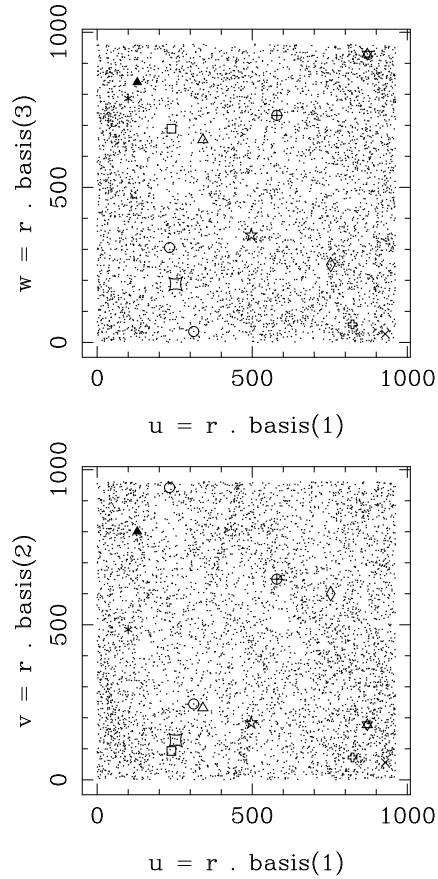
Figure 3. Same as for Fig. 2, except that the transformation has been rotated by an arbitrary angle, so that coincidences between objects should not be due to non-trivial topology.

if the transformation is due to a genuine physical identification.

An example of such an application is shown here, using a list of 5007 quasar positions (from the NASA/IPAC Extragalactic Database, NED), the highly luminous clusters discussed above, a few large superclusters, and the cold spots in the CMB tentatively attributed to density peaks (Cayón & Smoot 1995). The positions of all these objects are transformed to the fundamental cube and plotted in Fig. 2. Rotation of the fundamental cube by an arbitrary angle (i.e., no physical justification) gives a control sample in Fig. 3.

If identical quasars were seen as several different ghost images, then more close pairs should be seen in the map based on the cluster-derived fundamental cube than that based on the arbitrarily rotated fundamental cube. An excess of close pairs is not obvious to the eye, and a two-point correlation function (using 10 different control samples) confirms the lack of any statistical difference.

It could be the case that the angles of the fundamental cube are not perfect right angles, or that the sides are

of slightly unequal lengths, but in that case the quasars should still follow large scale structure, on a scale of about $50 - 150h^{-1}$ Mpc (e.g., de Lapparent et al. 1986; Geller & Huchra 1989; da Costa et al. 1993; Deng et al. 1996; Einasto et al. 1997). Again, the two-point correlation function shows no quantitative difference between structure in the case of the candidate fundamental cube and the control cases.

Note that the large scale variations in quasar density are due to the quasar catalogue containing a variety of observational catalogues over small areas of the sky, as well as large solid angle surveys. This is not a sign of multi-connectedness.

The group transformation indicated in Eq. 1 and Table 2 could also be applied to search for other copies of the "corners" of the fundamental polyhedron, each position at which another image of the (hypothetically single) cluster RXJ 1347.5-1145/Coma/CL 09104+4109 should be visible. Since Coma is nearly at the location of the observer, many of these images would be hidden behind one another. However, the four images at

$$
\begin{aligned}
\mathbf{r}_A &\equiv \mathbf{r}_{\text{Coma}} - \mathbf{r}.\mathbf{e}_1 = (0.392, 1^h 59^m, +18^\circ), \\
\mathbf{r}_B &\equiv \mathbf{r}_{\text{Coma}} - \mathbf{r}.\mathbf{e}_2 = (0.395, 20^h 32^m, -41^\circ), \\
\mathbf{r}_C &\equiv \mathbf{r}_{\text{Coma}} + \mathbf{r}.\mathbf{e}_3 = (0.406, 7^h 12^m, -43^\circ) \text{ and} \\
\mathbf{r}_D &\equiv \mathbf{r}_{\text{Coma}} - \mathbf{r}.\mathbf{e}_3 = (0.434, 18^h 31^m, 46^\circ)
\end{aligned}
\tag{2}
$$

[written as (z, α, δ)] would not be hidden. Firm constraints on the existence or non-existence of clusters at these positions, within uncertainties similar to the difference of the observed cluster triplet from an exact right angle of equal arm lengths (i.e., of order 1%), should be relatively easy to obtain observationally. Again, our closeness to Coma implies that the images should be seen at similar redshifts to those of RXJ 1347.5-1145 and CL 09104+4109, so the possibility that they are much fainter than these two clusters would require corresponding changes in luminosity over very short time intervals.

The lack of obvious candidate ghost images at these positions in the ROSAT All-Sky Survey (RASS) provides an additional argument against the candidate topology suggested.

However, for other candidate topologies which retain the identification of the three observed clusters, but are not simply T^3 with three equal fundamental lengths, ghost images would only have to be seen at positions $\mathbf{r}_A$ and $\mathbf{r}_B$, the near-"antipodes" of RXJ 1347.5-1145 and CL 09104+4109. RASS exposure in the direction of $\mathbf{r}_A$ is not deep enough to exclude a counterimage cluster and RXJ 203150.4-403656 is a plausible candidate at $\mathbf{r}_B$. So, spectroscopy to determine the redshift of the latter and an optical (or X-ray) search for a cluster within 2° of $(1^h 59^m, +18^\circ)$ and within $\delta(z) \lesssim 0.005$ of $z = 0.392$ would be needed to rule out variants on the candidate manifold suggested.

A final argument against the candidate manifold is the rarity of the cluster RXJ 1347.5-1145/Coma/-CL 09104+4109. According to the hypothesised manifold, this would be (historically) the brightest cluster in the Universe—and would happen to be just next door $(70h^{-1}$ Mpc away). The probability of the historically brightest cluster being this close to us in the Universe of total volume $(962h^{-1}$ Mpc$)^3$ is simply $P = [4\pi(70)^3/3]/(962)^3 = 0.0016$.

For a simply-connected topology, the brightest cluster is

RXJ 1347.5-1145 in its observed manifestation. As shown in Fig. 1, the probability of the occurrence of RXJ 1347.5-114 at this distance is quite high. Even the anthropic argument would not seem to be much help here.

4   CONCLUSION

Highly luminous X-ray clusters are a robust probe for finding ghost images of astrophysical objects which would reveal the possible non-triviality of the topology of the Universe. This is because observational statistics indicate that these clusters are likely to increase (or at a minimum retain the same) X-ray luminosity as time increases; while theoretically, gravity and conservation of matter imply that it is hard to see how the situation could be different for individual clusters.

Observations of successively brighter (more massive) rich clusters at higher redshifts implies successively greater volumes in which the Universe must be simply connected, while observation of the most X-ray bright (most massive) cluster at a redshift well below that to which virialised clusters are (eventually) discovered would be a clue to a multiconnected Universe. In the latter case, this brightest cluster would have ghost images among the population at higher redshifts.

The apparent lack of ghosts of RXJ 1347.5-1145 implies a lower limit to the size of the fundamental polyhedron, β, of about $1000h_{50}^{-1}$ Mpc (for $\Omega_0 = 1, \lambda_0 = 0$). This limit could be doubled in size, without having to observe to fainter flux limits than those of the ROSAT All-Sky Survey, by a survey through the galactic plane in hard X-rays, if some means of confirming which sources are clusters could be found (e.g., by surface brightness profiles combined with spectral shapes).

The consideration of highly luminous X-ray clusters has indicated what is geometrically an exciting candidate identification of ghost images. RXJ 1347.5-1145, the Coma cluster and the cluster CL 09104+4109 together form what is nearly a right angle of nearly equal arm lengths, just what would be expected for a hypertoroidal geometry (for flat curvature). The inferred transformation from the covering space to the fundamental cube is presented in Table 2 and Eqn 1. However, the local physical properties of these three clusters do not seem to support identity; CMB constraints against hypertoroidal topologies have been well studied; a mapping of ≈ 5000 quasar three-dimensional positions into the implied fundamental cube of the hypertoroidal manifold does not support identity; not all of the expected four extra ghost images of RXJ 1347.5-1145 and CL 09104+4109 are obvious; and the probability that we are as close as we are to the historically brightest cluster in the Universe RXJ 1347.5-1145/Coma/CL 09104+4109 would be only 0.16%.

Nevertheless, observations to determine the redshift of RXJ 203150.4-403656 and to search for a cluster within 2° of $(1^h 59^m, +18^\circ)$ and $\delta(z) \lesssim 0.005$ of $z = 0.392$ would be useful to rule out (or detect!) variants on the candidate manifold suggested.

5   ACKNOWLEDGEMENTS

REFERENCES

Allen, S.W., Fabian, A.C., Edge, A.C., Bautz, M.W., Furuzawa, A., Tawara, Y., 1996, MNRAS, 283, 263 (astro-ph/9609201)

Briel, U.G., Henry, J.P., Böhringer, H., 1992, A.&A., 259, L31

Bucher, M., Goldhaber, A.S., 1995, Phys.Rev.D, 52, 3314

Cavaliere, A., Padovani, P., 1988, Ap.J., 333, L33

Cayón, L., Smoot, G., 1995, Ap.J., 452, 487

Cornish, N.J., Spergel, D.N., Starkman, G.D., 1996, Phys.Rev.-Lett., 77, 215

Cornish, N.J., Spergel, D.N., Starkman, G.D., 1997, gr-qc/9602039

da Costa, L.N., in Cosmic Velocity Fields, ed. Bouchet, F., Lachièze-Rey, M., (Gif-sur-Yvette, France: Editions Frontières), p475

de Lapparent, V., Geller, M.J., Huchra, J.P., 1986, Ap.J., 302, L1

Deng, X.-F., Deng, Z.-G., Xia, X.-Y., 1996, Chin.Astron.Astroph., 20, 383

de Oliveira Costa, A., Smoot, G.F., 1995, ApJ, 448, 477

de Sitter, W., 1917, MNRAS, 78, 3

Ebeling, H., et al., 1996a, in Proc. Röntgenstrahlung from the Universe, ed. Zimmermann, H.U. & Trümper, J., MPE Report 263 (astro-ph/9512104)

Ebeling, H., Edge, A.C., Fabian, A.C., Allen, S.W., Crawford, C.S., Böhringer, H., 1997, Ap.J., 20 April,

Ebeling, H., Voges, W., Böhringer, H., Edge, A.C., 1993, A.&A., 275, 360

Ebeling, H., Voges, W., Böhringer, H., Edge, A.C., Huchra, J.P., Briel, U.G., 1996b, MNRAS, 281, 799

Edge, A.C., Stewart, G.C., Fabian, A.C., Arnaud, K.A., 1990, MNRAS, 245, 559

Edge, A.C., Stewart, G.C., 1991, MNRAS, 252, 414

Einasto, J., et al., 1997, Nature, 385, 139

Elbaz, D., Arnaud, M., Böhringer, H., 1995, A.&A., 293, 337

Evrard, A.E., Metzler, C.A., Navarro, J.F., 1996, Ap.J., 469, 494

Fabian, A.C., Crawford, C.S., 1995, MNRAS, 274, L63

Fagundes, H.V., 1985, Ap.J., 291, 450

Fagundes, H.V., 1989, Ap.J., 338, 618

Fagundes, H.V., 1996, Ap.J., 470, 43

Fagundes, H.V., Wichoski, U.F., 1987, Ap.J., 322, L5

García Vargs, M.L., Díaz, A., Terlevich, R., Terlevich, E., 1993, Astrophys.Sp.Sci., 205, 77

Geller, M.J., Huchra, J.P., 1989, Science, 246, 897

Gioia, J.M., et al., 1990, Ap.J.Supp., 72, 567

Gioia, J.M., Luppino, G.A., 1994, Ap.J.Supp., 94, 583

Gott, J.R.I., 1980, MNRAS, 193, 153

Gott, J.R., 1982, Nature, 295, 304

Gott, J.R., 1986, in Inner Space/Outer Space, eds E.W. Kolb et al. (Chicago: Univ. Chicago Press), p362

Hall, P.B., Ellingson, E., Green, R.F., 1997, A.J., 113, 1179 (astro-ph/9612241)

Hawking, S., 1984, NuclPhysB, 239, 257

Henry, J.P., Arnaud, K.A., 1991, Ap.J., 372, 410

Henry, J.P., et al., 1992, Ap.J., 386, 408

Jing, Y.-P., Fang, L.-Z., 1994, astro-ph/9409072

Lachièze-Rey, M., Luminet, J.-P., 1995, PhysRep, 254, 136

Lehoucq, R., Luminet, J.-P., Lachièze-Rey, M., 1996, A.&A., 313, 339

Lemaître, G., 1958, in La Structure et l'Evolution de l'Univers, Onzième Conseil de Physique Solvay, ed. Stoops, R., (Brussels: Stoops), p1

Linde, A., 1995, Phys.Lett.B, 351, 99

Luppino, G.A., Gioia, I.M., 1995, Ap.J., 445, L77

Orwell, G., 1945, Animal Farm (UK: Martin Secker & Warburg)

Oukbir, J., Bartlett, J.G., Blanchard, A., 1997, A.&A., 320, 365 (astro-ph/9611089)

Oukbir, J., Blanchard, A., 1997, A.&A., 317, 1

Ratra, B., Peebles, P.J.E., 1994, Ap.J., 432, L5

Roukema, B.F., 1996, MNRAS, 283, 1147

Sasaki, M., Tanaka, T., Yamamoto, K., Yokoyama, J., 1993, Prog.Theor.Phys., 90, 1019

Schechter, P., 1976, Ap.J., 203, 297

Schindler, S., Hattori, M., Neumann, D.M., Böhringer, H., 1996, astro-ph/9603037

Scoville, N., Soifer, B.T., 1991, in Massive Stars in Starbursts (Britain: Camb.Univ.Press)

Sokolov, D.D., Shvartsman, V.F., 1974, Sov.Phys.JETP, 39, 196

Starkman, G.D., Cornish, N.J., Spergel, D.N., private communication, 1996

Starobinsky, A.A., 1993, JETPLett, 57, 622

Steidel, C.C., Giavalisco, M., Pettini, M., Dickinson, M. & Adelberger, K.L., 1996, astro-ph/9602024

Stevens, D., Scott, D., Silk, J., 1993, PhysRevLett, 71, 20

Terlevich, R., Melnick, J., 1985, MNRAS, 213, 841

Thurston, W.P., Weeks, J.R., 1984, ScAmer, July, 94

Zel'dovich, Y.B., Grishchuk, L.P., 1984, MNRAS, 207, 23P